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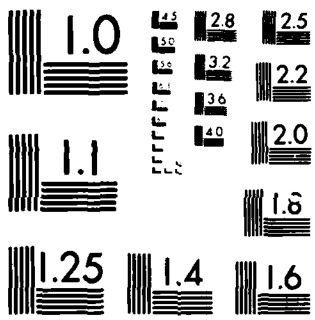
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AFOSR-TR- 86-0035

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RELIABILITY IMPORTANCE FOR CONTINUUM STRUCTURE FUNCTIONS*

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*Research supported by the Air Force Office of Scientific Research, AFSC, USAF, under grant AFOSR-84-0243. The US Government is authorised to reproduce and distribute reprints for Governmental purposes notwithstanding any copyright notation thereon.

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ABSTRACT

A continuum structure function is a nondecreasing mapping from the unit hypercube to the unit interval. A definition of the reliability importance, $R_i(\alpha)$ say, of component i at level α $(0<\alpha\leq 1)$ is proposed. Some properties of this function are deduced, in particular conditions under which $\lim_{\alpha\to 0} R_i(\alpha) = \lim_{\alpha\to 1} R_i(\alpha) = 0$ and conditions under which $R_i(\alpha)$ is positive $(0<\alpha<1)$.

KEYWORDS: Continuum structure function; reliability importance; key vector

A-1

1. INTRODUCTION

Let ϕ: $\{0,1\}^n \mapsto \{0,1\}$ be a binary coherent structure function and let h: $[0,1]^n \mapsto [0,1]$ be the corresponding reliability function (see Barlow and Proschan (1975a), Chapters 1 and 2). The reliability importance of component i is defined as

$$I(i) = \frac{\partial h(\underset{\sim}{p})}{\partial p_i} = h(1_i,\underset{\sim}{p}) - h(0_i,\underset{\sim}{p})$$

$(i=1,2,\ldots,n)$, writing $(\beta_i,\underset{\sim}{p}) = (p_1,\ldots,p_{i-1},\beta,p_{i+1},\ldots,p_n)$ where $p_i = P\{X_i=1\}$ and where $X_1,\ldots,X_n$ are independent binary random variables denoting the states of the components of ϕ. This definition is due to Birnbaum (1969); see Barlow and Proschan (1975b) and Natvig (1979), (1984) for some alternative approaches. Various authors have proposed extensions of this concept to the multistate case (e.g. Barlow and Wu (1978), Griffith (1980), Natvig (1982), Block and Savits (1982)), but a general theory of reliability importance for structure functions on domains other than $\{0,1\}^n$ has yet to be developed. In this paper, we present a definition of reliability importance for <u>continuum structure functions</u> (CSFs), i.e. mappings of the form γ: $\Delta \mapsto [0,1]$, where $\Delta = [0,1]^n$, which are nondecreasing in each argument and which satisfy $\gamma(\underset{\sim}{0}) = 0$ and $\gamma(\underset{\sim}{1}) = 1$ where $\underset{\sim}{\beta}$ denotes $(\beta,\ldots,\beta)$. See Block and Savits (1984) and Baxter (1984), (1986) for further details of CSFs. The reliability importance, $R_i(\alpha)$ say, of component i $(i=1,2,\ldots,n)$ will depend on the state α $(0\leq\alpha\leq 1)$ of the system. Our main results are conditions on γ under which $\lim_{\alpha\to 0} R_i(\alpha) = \lim_{\alpha\to 1} R_i(\alpha) = 0$ and conditions under which $R_i(\alpha)$ is positive.

We shall make frequent use of the following sets:

$$U_\alpha = \{\underset{\sim}{x}\in\Delta \mid \gamma(\underset{\sim}{x})\geq\alpha\}$$

$$L_\alpha = \{\underset{\sim}{x}\in\Delta \mid \gamma(\underset{\sim}{x})\leq\alpha\}$$

$$P_\alpha = \{\underset{\sim}{x}\in\Delta \mid \gamma(\underset{\sim}{x})\geq\alpha \text{ whereas } \gamma(\underset{\sim}{y})<\alpha \text{ for all } \underset{\sim}{y}<\underset{\sim}{x}\}$$

$$K_\alpha = \{\underset{\sim}{x}\in\Delta \mid \gamma(\underset{\sim}{x})\leq\alpha \text{ whereas } \gamma(\underset{\sim}{y})>\alpha \text{ for all } \underset{\sim}{y}>\underset{\sim}{x}\}$$

where $\underset{\sim}{y}<(>)\underset{\sim}{x}$ means that $\underset{\sim}{y} \leq (\geq)\underset{\sim}{x}$ but that $\underset{\sim}{y} \neq \underset{\sim}{x}$.

2. KEY VECTORS

The motivation for our definition (below) is most readily understood by observing that one can write

$$I(i) = P\{\phi(\underset{\sim}{X})=1 \mid X_i=1\} - P\{\phi(\underset{\sim}{X})=1 \mid X_i=0\},$$

i.e. $I(i)$ is the probability that repairing component i will restore a failed system to the operating state (or, equivalently, that the failure of component i will cause an operating system to fail). A possible generalisation of $I(i)$ to the continuum case would be to regard part of the unit interval, say $[0,\alpha)$ $(0<\alpha\leq 1)$, as corresponding to the failure states of the system and to regard $[\alpha,1]$ as the operating states, in which case one could define the reliability importance of component i $(i=1,2,\ldots,n)$ to be

$$P\{\gamma(\underset{\sim}{X})\geq\alpha \mid X_i\geq\alpha\} - P\{\gamma(\underset{\sim}{X})\geq\alpha \mid X_i<\alpha\}.$$

Consideration of the CSF $\gamma(x_1,x_2) = x_1x_2$ suggests that this definition is not wholly satisfactory: if $x_1 = x_2 = \beta \in [\alpha,\sqrt{\alpha})$ $(0<\alpha<1)$, then neither component is in the failed state even though the system itself should be regarded as failed. This difficulty may be circumvented by replacing $\underset{\sim}{\alpha}$ by a suitably chosen element of ∂U_α; considerations of symmetry indicate that the vector chosen, called the key vector, should also lie on the diagonal of the unit hypercube. Hence, before proceeding to a definition of reliability importance for CSFs, it is convenient to define and study the key vector of U_α.

Definition

Let $H = \{\underset{\sim}{\alpha} \mid \underset{\sim}{0} \le \underset{\sim}{\alpha} \le \underset{\sim}{1}\}$ be the diagonal of the unit hypercube. We say that the vector $\underset{\sim}{\delta} = \underset{\sim}{\delta}(\alpha) = H \cap \partial U_\alpha$ is the key vector of U_α and we call δ the key element.

Lemma 2.1

The CSF γ is right (left)-continuous if and only if each $U_\alpha(L_\alpha)$ is closed.

Proof: A CSF is right (left)-continuous if and only if it is upper (lower) semicontinuous which is the case if and only if each $U_\alpha^c(L_\alpha^c)$ is open (Royden (1968), p. 161). □

Theorem 2.2

For any CSF γ, the key vector always exists and, if γ is continuous, $\gamma(\underset{\sim}{\delta}) = \alpha$ for all $\alpha \in (0,1]$.

Proof: To show that the key vector exists for any CSF, it is sufficient to show that $H \cap \partial U_\alpha \neq \emptyset$ for all $\alpha \in (0,1]$.

Let γ be an arbitrary CSF. Then $\underset{\sim}{1} \in U_\alpha$ for all $\alpha \in (0,1]$ by definition, so $\bar{U}_\alpha \neq \emptyset$ for all $\alpha \in (0,1]$. If $\partial U_\alpha = \bar{U}_\alpha$, it is immediate that $H \cap \partial U_\alpha \neq \emptyset$ since $\underset{\sim}{1} \in H \cap \bar{U}_\alpha$ for all $\alpha \in (0,1]$. Suppose that ∂U_α is a proper subset of $\bar{U}_\alpha$ and consider $\Delta' = \Delta - \partial U_\alpha$. Since $\bar{U}_\alpha^c$ and $\bar{U}_\alpha$ are disjoint and ∂U_α is a proper subset of $\bar{U}_\alpha$, it follows that $\Delta' = (\bar{U}_\alpha^c \cup \bar{U}_\alpha) - \partial U_\alpha = \bar{U}_\alpha^c \cup (\bar{U}_\alpha - \partial U_\alpha)$ is a separation of Δ', i.e. Δ' is a disconnected set. Now suppose that $H \cap \partial U_\alpha = \emptyset$ for some $\alpha \in (0,1]$. Then, for all $\underset{\sim}{\beta} \in H$, $\underset{\sim}{\beta} \notin \partial U_\alpha$, so $H \subset \Delta'$. Clearly, H is connected. Since Δ' is a disconnected set with separation $\bar{U}_\alpha^c \cup (\bar{U}_\alpha - \partial U_\alpha)$, H must be properly contained in either $\bar{U}_\alpha^c$ or $\bar{U}_\alpha - \partial U_\alpha$. Since ∂U_α is a proper subset of $\bar{U}_\alpha$ and since $\underset{\sim}{1} \in \bar{U}_\alpha$, it is obvious that $\underset{\sim}{1} \notin \partial U_\alpha$, i.e. $\underset{\sim}{1} \in \bar{U}_\alpha - \partial U_\alpha$, and thus $H \cap (\bar{U}_\alpha - \partial U_\alpha) \neq \emptyset$. Further, since $\alpha \in (0,1]$, $\underset{\sim}{0} \in \bar{U}_\alpha^c$, so $H \cap \bar{U}_\alpha^c \neq \emptyset$. This is a contradiction to the assertion that H must be properly contained in either $\bar{U}_\alpha^c$ or $\bar{U}_\alpha - \partial U_\alpha$. Thus $H \cap \partial U_\alpha \neq \emptyset$ for all $\alpha \in (0,1]$ for any CSF, as claimed.

We now show that if γ is continuous, then $\gamma(\underset{\sim}{\delta}) = \alpha$. Since, by continuity, $\underset{\sim}{\delta} \in \partial U_\alpha \subset U_\alpha$, it follows that $\gamma(\underset{\sim}{\delta}) \geq \alpha$. Suppose that there exists an $\alpha \in (0,1]$ such that $\gamma(\underset{\sim}{\delta}) > \alpha$. Since $\underset{\sim}{\delta} \in \partial U_\alpha$, for any $\varepsilon > 0$ and for all n we have $\underset{\sim}{\delta} - 2^{-n}\underset{\sim}{\varepsilon} \in H$ whereas $\underset{\sim}{\delta} - 2^{-n}\underset{\sim}{\varepsilon} \notin \partial U_\alpha$, so $\underset{\sim}{\delta} - 2^{-n}\underset{\sim}{\varepsilon} \in U_\alpha^c$, i.e. $\underset{\sim}{\delta} - 2^{-n}\underset{\sim}{\varepsilon} \in L_\alpha$. However, $\lim_{n\to\infty} (\underset{\sim}{\delta} - 2^{-n}\underset{\sim}{\varepsilon}) \notin L_\alpha$ so L_α is not closed. Hence, by Lemma 2.1, γ is not continuous. This is a contradiction and so $\gamma(\underset{\sim}{\delta}) = \alpha$ as claimed.

This completes the proof. $\square$

Since the key vector $\underset{\sim}{\delta}$ exists for any CSF and for any $\alpha \in (0,1]$, and since Δ is symmetric about H, we define reliability importance as follows.

Definition

The reliability importance $R_i(\alpha)$ of component i at level $\alpha \in \text{Im } \gamma - \{0\}$ for the CSF γ is defined as

$$R_i(\alpha) = P\{\gamma(\underset{\sim}{X}) \geq \alpha | X_i \geq \delta\} - P\{\gamma(\underset{\sim}{X}) \geq \alpha | X_i < \delta\}$$

where $\underset{\sim}{X}$ is a random vector and where δ is the key element of U_α. (Im γ denotes the image of γ.)

Remarks

1. We may interpret $R_i(\alpha)$ as $P\{\gamma(\underset{\sim}{X}) \geq \alpha$ iff $X_i \geq \delta\}$.

2. Replacing Δ and γ by $\{0,1\}^n$ and ϕ, a binary coherent structure function, respectively, in this definition yields the (Birnbaum) reliability importance of component i, and hence the above definition is a direct generalisation of reliability importance in the binary case.

3. BOUNDARY BEHAVIOUR

In this section, we derive conditions under which $\lim_{\alpha \to 0} R_i(\alpha) = \lim_{\alpha \to 1} R_i(\alpha) = 0$, i.e. under which component i does not affect the state of the system when the latter is at one of the extrema of its range. The following notation will subsequently prove useful:

$$U(\underset{\sim}{y}) = \{\underset{\sim}{x}\in\Delta \mid \underset{\sim}{x} \geq \underset{\sim}{y}\}$$

$$L(\underset{\sim}{y}) = \{\underset{\sim}{x}\in\Delta \mid \underset{\sim}{x} < \underset{\sim}{y}\}$$

$$f_i(\alpha) = P\{\gamma(\underset{\sim}{X}) \geq \alpha \mid X_i \geq \delta\}$$

$$g_i(\alpha) = P\{\gamma(\underset{\sim}{X}) \geq \alpha \mid X_i < \delta\}$$

where $\underset{\sim}{X}$ is a random vector so that $R_i(\alpha) = f_i(\alpha) - g_i(\alpha)$.

<u>Lemma 3.1</u>

Let γ be a continuous CSF and write $P_\alpha = \{\underset{\sim}{y}_t, t\in T(\alpha)\}$. Then

$$U_\alpha = \bigcup_{t\in T(\alpha)} U(\underset{\sim}{y}_t).$$

<u>Proof</u>: See Block and Savits (1984), Theorem 2.

<u>Proposition 3.2</u>

For any CSF γ,

(i) $\lim\limits_{\alpha\to 0} U_\alpha = A_0$ where $A_0 = \{\underset{\sim}{x}\in\Delta \mid \gamma(\underset{\sim}{x})>0\}$

(ii) $\lim\limits_{\alpha\to 1} U_\alpha = A_1$ where $A_1 = \{\underset{\sim}{x}\in\Delta \mid \gamma(\underset{\sim}{x})=1\}$.

<u>Proof</u>: Since γ is nondecreasing, $U_\alpha \supset U_\beta$ whenever $\alpha < \beta$.
(i) For given $\alpha \in (0,1)$, let N be a positive integer satisfying $\frac{1}{N} \leq \alpha$. Further, let $\alpha > \alpha_1 > \alpha_2 > \cdots > 0$ be a refinement of $[0,\alpha)$ where $\alpha_m = 1/(N+m)$. Then the sequence $\{U_{1/(N+m)}\}_{m=1}^{\infty}$ is increasing with limit

$\lim_{\alpha\to 0} U_\alpha = \lim_{m\to\infty} U_{1/(N+m)} = \bigcup_{m=1}^{\infty} U_{1/(N+m)}$. We show that $\bigcup_{m=1}^{\infty} U_{1/(N+m)} = A_0$.

Let $\underset{\sim}{x} \in \bigcup_{m=1}^{\infty} U_{1/(N+m)}$; then $\underset{\sim}{x} \in U_{1/(N+m)}$ for some m so that $\gamma(\underset{\sim}{x}) \geq 1/(N+m) > 0$, i.e. $\underset{\sim}{x} \in A_0$, and hence $\bigcup_{m=1}^{\infty} U_{1/(N+m)} \subset A_0$. Conversely, let $\underset{\sim}{x} \in A_0$. Then $\gamma(\underset{\sim}{x}) = \beta$ for some $\beta > 0$ and there exists an integer N' such that $\frac{1}{N'} \leq \beta$ and an integer m such that $N+m \geq N'$ so $\gamma(\underset{\sim}{x}) = \beta \geq \frac{1}{N'} \geq 1/(N+m)$ and $\underset{\sim}{x} \in U_{1/(N+m)}$, hence $A_0 \subset \bigcup_{m=1}^{\infty} U_{1/(N+m)}$.

(ii) The proof is similar. $\square$

Theorem 3.3

Suppose that γ is a continuous CSF and that $X_1,\ldots,X_n$ are independent, absolutely continuous random variables.

(i) If for all $\underset{\sim}{y} \in P_1$, $y_j = 1$ for some $j \neq i$, then $\lim_{\alpha\to 1} R_i(\alpha) = 0$.

(ii) If for all $\underset{\sim}{w} \in K_0$, $w_j = 0$ for some $j \neq i$, then $\lim_{\alpha\to 0} R_i(\alpha) = 0$.

Proof: Since γ is nondecreasing, for any $\alpha \in (0,1]$

$$P\{\underset{\sim}{X}\in U_\alpha | X_i=0\} < f_i(\alpha) \leq P\{\underset{\sim}{X}\in U_\alpha | X_i=1\}$$

and

$$P\{\underset{\sim}{X}\in U_\alpha | X_i=0\} < g_i(\alpha) \leq P\{\underset{\sim}{X}\in U_\alpha | X_i=1\}.$$

Thus, if we show that $\lim_{\alpha\to 1} P\{\underset{\sim}{X}\in U_\alpha | X_i=1\} = 0$ under the hypothesis of (i), then $\lim_{\alpha\to 1} f_i(\alpha) = \lim_{\alpha\to 1} g_i(\alpha) = 0$ so that $\lim_{\alpha\to 1} R_i(\alpha) = 0$. Further, if we show that $\lim_{\alpha\to 0} P\{\underset{\sim}{X}\in U_\alpha | X_i=0\} = 1$ under the hypothesis of (ii), then

$\lim_{\alpha\to 0} f_i(\alpha) = \lim_{\alpha\to 0} g_i(\alpha) = 1$ so that $\lim_{\alpha\to 0} R_i(\alpha) = 0$.

(i) Since, by Proposition 3.2, $\lim_{\alpha\to 1} U_\alpha = A_1 = \{\underset{\sim}{x}\in\Delta | \gamma(\underset{\sim}{x})=1\}$, it follows from the continuity of probability measures that $\lim_{\alpha\to 1} P\{\underset{\sim}{X}\in U_\alpha | X_i=1\} =$

$P\{\underset{\sim}{X}\in A_1 | X_i=1\}$. We show that $P\{\underset{\sim}{X}\in A_1 | X_i=1\} = 0$.

Define $P_1^j = \{\underset{\sim}{y}\in P_1 | y_j=1,\ j\neq i\}$ and write $P_1^j = \{\underset{\sim}{y}_\tau, \tau\in T(j)\}$; by hypothesis, the P_1^j's $(j\neq i)$ form a partition of P_1. Define

$A_j = \bigcup_{\tau\in T(j)} U(\underset{\sim}{y}_\tau)$, $j\neq i$. Then, by Lemma 3.1, $A_1 = U_1 = \bigcup_{j\neq i} A_j$, so

$$P\{\underset{\sim}{X}\in A_1 | X_i=1\} = P\{\underset{\sim}{X}\in \bigcup_{j\neq i} A_j | X_i=1\}.$$

By the inclusion-exclusion principle,

$$P\{\underset{\sim}{X}\in A_1 | X_i=1\} = \sum_{\ell=1}^{n-1} (-1)^{\ell-1}\pi_\ell$$

where $\pi_\ell = \sum_{\substack{1\le k_1<k_2<\cdots<k_\ell\le n-1 \\ k_j\neq i \text{ for } j=1,2,\ldots,\ell}} P\{\underset{\sim}{X}\in A_{k_1}\cap A_{k_2}\cap\cdots\cap A_{k_\ell} | X_i=1\}.$

We show that $\pi_1 = 0$.

By definition, $\pi_1 = \sum_{j\neq i} P\{\underset{\sim}{X}\in A_j | X_i=1\}$.

Let $z_q = \inf_{\tau\in T(j)} y_{\tau q}$, $q\neq j$, $q=1,2,\ldots,n$ and $\xi_j = \min(z_1,\ldots,z_{j-1},z_{j+1},\ldots,z_n)$,

and let $Q_j = [\xi_j,1]\times\cdots\times[\xi_j,1]\times\{1\}_j\times[\xi_j,1]\times\cdots\times[\xi_j,1]$ where the subscript j on $\{1\}$ indicates that this is the j^{th} term in the product. We claim that $A_j \subset Q_j$. Let $\underset{\sim}{x} \in A_j$. Then $\underset{\sim}{x} \in U(\underset{\sim}{y}_\tau)$ for some $\tau \in T(j)$, and hence $\underset{\sim}{x} \geq \underset{\sim}{y}_\tau$ and $y_j = 1$, so that $x_j = 1$ and $x_q \geq z_q$ for $q=1,2,\ldots,n$, $q \neq j$. Thus $x_j = 1$ and $x_q \geq \xi_j$ for $q=1,2,\ldots,n$, $q \neq j$, from which it follows that $\underset{\sim}{x} \in Q_j$. This holds for all $\underset{\sim}{x} \in A_j$, so $A_j \subset Q_j$. Hence

$$\pi_1 = \sum_{j \neq i} P\{\underset{\sim}{X} \in A_j \mid X_i = 1\}$$

$$\leq \sum_{j \neq i} P\{\underset{\sim}{X} \in Q_j \mid X_i = 1\}$$

$$= \sum_{j \neq i} P\{(1_i, \underset{\sim}{X}) \in Q_j\}$$

$$= \sum_{j \neq i} P\{X_1 \geq \xi_j, \ldots, X_{j-1} \geq \xi_j, X_j = 1, X_{j+1} \geq \xi_j, \ldots, X_n \geq \xi_j\}$$

$$= \sum_{j \neq i} \prod_{k \neq j} P\{X_k \geq \xi_j\} P\{X_j = 1\} \text{ by independence}$$

$$= 0 \text{ since each } X_j \text{ is absolutely continuous,}$$

so $\pi_1 = 0$ as claimed.

Since, for any $\ell \geq 2$, $\pi_\ell \leq \pi_1 = 0$, we see that $P\{\underset{\sim}{X} \in A_1 \mid X_i = 1\} = 0$ as claimed.

(ii) The proof is similar. $\square$

4. A CONDITION FOR POSITIVE RELIABILITY IMPORTANCE

In this section, we derive a condition under which the reliability importance $R_i(\alpha)$ is positive for $\alpha \in (0,1)$.

Lemma 4.1

Let ν be Lebesgue measure on $\mathbb{R}^n$. Then

(i) $\nu\{U(\underset{\sim}{y})\} = 0$ if and only if $y_i = 1$ for some $i=1,2,\dots,n$

$\nu\{L(\underset{\sim}{y})\} = 0$ if and only if $y_i = 0$ for some $i=1,2,\dots,n$

(ii) $\nu\{\bigcup_{t\in T} U(\underset{\sim}{y}_t)\} = 0$ if and only if $\nu\{U(\underset{\sim}{y}_t)\} = 0$ for all $t \in T$

$\nu\{\bigcup_{t\in T} L(\underset{\sim}{y}_t)\} = 0$ if and only if $\nu\{L(\underset{\sim}{y}_t)\} = 0$ for all $t \in T$

where T is an index set.

Proof: (i) This is trivial.

(ii) Suppose that $\nu\{\bigcup_{t\in T} U(\underset{\sim}{y}_t)\} = 0$ and that, conversely, there exists some $t' \in T$ such that $\nu\{U(\underset{\sim}{y}_{t'})\} > 0$. Then $\nu\{\bigcup_{t\in T} U(\underset{\sim}{y}_t)\} \geq \nu\{U(\underset{\sim}{y}_{t'})\} > 0$, a contradiction.

Suppose, now, that $\nu\{U(\underset{\sim}{y}_t)\} = 0$ for all $t \in T$. Let $\Delta'' = \Delta - (0,1)^n$; clearly $\nu\{\Delta''\} = 0$. Let $\underset{\sim}{x} \in \bigcup_{t\in T} U(\underset{\sim}{y}_t)$; then $\underset{\sim}{x} \in U(\underset{\sim}{y}_t)$ for some $t \in T$. Since $\nu\{U(\underset{\sim}{y}_t)\} = 0$ for all $t \in T$, it follows from (i) that $y_i = 1$ for some $i=1,2,\dots,n$. Thus, by definition of $U(\underset{\sim}{y}_t)$, $\underset{\sim}{x} \geq \underset{\sim}{y}_t$ implies $x_i = 1$, i.e. $\underset{\sim}{x} \in \Delta''$. This holds for all $\underset{\sim}{x} \in \bigcup_{t\in T} U(\underset{\sim}{y}_t)$, so $\bigcup_{t\in T} U(\underset{\sim}{y}_t) \subset \Delta''$ and hence $\nu\{\bigcup_{t\in T} U(\underset{\sim}{y}_t)\} = 0$.

A similar argument shows that $\nu\{\bigcup_{t \in T} L(\underset{\sim}{y}_t)\} = 0$ if and only if $\nu\{L(\underset{\sim}{y}_t)\} = 0$ for all $t \in T$. $\square$

Theorem 4.2

The distribution function F is absolutely continuous if and only if $\mu << \nu$ where ν is Lebesgue measure and where μ is the induced Lebesgue-Stieltjes measure satisfying $\mu\{(-\infty,x]\} = F(x)$ for all $x \in \mathbb{R}$.

Proof: See Billingsley (1979, p. 367).

Proposition 4.3

Let γ be a continuous CSF. If $\nu\{U_\alpha\} > 0$ for $\alpha \in (0,1)$, then $\delta \in (0,1)$ where δ is the key element of U_α.

Proof: We show that $\delta \notin \{0,1\}$. Suppose that $\delta = 0$ for $\alpha \in (0,1)$. Since γ is continuous, $\underset{\sim}{\delta} = \underset{\sim}{0} \in \partial U_\alpha \subset U_\alpha$, and so $\gamma(\underset{\sim}{0}) \geq \alpha > 0$, a contradiction to the definition of γ.

Suppose, now, that $\delta = 1$ for $\alpha \in (0,1)$ and let $\Delta'' = \Delta - (0,1)^n$. We show that $U_\alpha \subset \Delta''$. It is sufficient to show that for all $\underset{\sim}{x} \in U_\alpha$, $x_i = 1$ for some $i=1,2,\ldots,n$. Suppose, conversely, that there exists a vector $\underset{\sim}{x} \in U_\alpha$ such that $x_i < 1$ for all $i=1,2,\ldots,n$. Then $\xi = \max(x_1,\ldots,x_n) < 1$ and $\underset{\sim}{\xi} \in H \cap U_\alpha$, in contradiction to the assumption that $\underset{\sim}{\delta} = \underset{\sim}{1} \in H \cap \partial U_\alpha$. Hence, for all $\underset{\sim}{x} \in U_\alpha$, $x_i = 1$ for some $i=1,2,\ldots,n$ so that $U_\alpha \subset \Delta''$. Since $\nu\{\Delta''\} = 0$, we see that $\nu\{U_\alpha\} = 0$, a contradiction to the given hypothesis. $\square$

We introduce the following notation for future reference. Let

$$D_i = D_i(\delta) = [0,1]\times\cdots\times[0,1]\times[\delta,1]_i\times[0,1]\times\cdots\times[0,1]$$

$$E_i = E_i(\delta) = [0,1]\times\cdots\times[0,1]\times[0,\delta)_i\times[0,1]\times\cdots\times[0,1]$$

where the subscript i labels the i^{th} term in the product.

Theorem 4.4

Let γ be a continuous CSF such that $\nu\{U_\alpha\} > 0$ for all $\alpha \in (0,1)$ where ν is Lebesgue measure on $\mathbb{R}^n$ and suppose that $X_1,\ldots,X_n$ are independent, absolutely continuous random variables. Then $R_i(\alpha) = 0$ for $\alpha \in (0,1)$ *if and only if* $y_i = 0$ for every $\underset{\sim}{y} \in P_\alpha$ for which $\nu\{U(\underset{\sim}{y})\} > 0$.

Proof: Define the induced Lebesgue-Stieltjes measure $P_{\underset{\sim}{X}} = P\circ\underset{\sim}{X}^{-1}$. Observe that, since, from Proposition 4.3, the key element $\delta \in (0,1)$, and since X_i is absolutely continuous, it follows from Theorem 4.2 that $P\{X_i \geq \delta\} > 0$ and $P\{X_i < \delta\} > 0$. Write $P_\alpha = \{\underset{\sim}{y}_t, t\in T(\alpha)\}$. Then, from Lemma 3.1, $U_\alpha = \bigcup_{t\in T(\alpha)} U(\underset{\sim}{y}_t)$; this is clearly a Borel set and so we can write

$$f_i(\alpha) = P_{\underset{\sim}{X}}\{\bigcup_{t\in T(\alpha)} U(\underset{\sim}{y}_t) \cap D_i\}/P\{X_i \geq \delta\}$$

$$g_i(\alpha) = P_{\underset{\sim}{X}}\{\bigcup_{t\in T(\alpha)} U(\underset{\sim}{y}_t) \cap E_i\}/P\{X_i < \delta\}.$$

"If" Define $P_{\alpha 1} = \{\underset{\sim}{y}\in P_\alpha | y_i \neq 1 \text{ for all } i=1,2,\ldots,n\}$,

$P_{\alpha 2} = \{\underset{\sim}{y}\in P_\alpha | y_i = 1 \text{ for some } i=1,2,\ldots,n\}$

and write $P_{\alpha 1} = \{\underset{\sim}{y}_\ell, \ell \in L(\alpha)\}$ and $P_{\alpha 2} = \{\underset{\sim}{y}_s, s \in S(\alpha)\}$ for suitable index sets $L(\alpha)$ and $S(\alpha)$. Then, from Lemma 4.1(i), $\nu\{U(\underset{\sim}{y}_\ell)\} > 0$ for all $\ell \in L(\alpha)$ and $\nu\{U(\underset{\sim}{y}_s)\} = 0$ for all $s \in S(\alpha)$, so, from Lemma 4.1 (ii),

$$\nu\{\bigcup_{s\in S(\alpha)} U(\underset{\sim}{y}_s)\} = 0. \tag{4.1}$$

Now

$$P_{\underset{\sim}{X}}\{\bigcup_{t\in T(\alpha)} U(\underset{\sim}{y}_t) \cap D_i\}$$

$$= P_{\underset{\sim}{X}}\{[\bigcup_{\ell\in L(\alpha)} U(\underset{\sim}{y}_\ell) \cup \bigcup_{s\in S(\alpha)} U(\underset{\sim}{y}_s)] \cap D_i\}$$

$$= P_{\underset{\sim}{X}}\{\bigcup_{\ell\in L(\alpha)} U(\underset{\sim}{y}_\ell) \cap D_i\} + P_{\underset{\sim}{X}}\{\bigcup_{s\in S(\alpha)} U(\underset{\sim}{y}_s) \cap D_i\}$$

$$- P_{\underset{\sim}{X}}\{\bigcup_{\ell\in L(\alpha)} U(\underset{\sim}{y}_\ell) \cap \bigcup_{s\in S(\alpha)} U(\underset{\sim}{y}_s) \cap D_i\}.$$

Consider the second term in this sum; clearly

$$P_{\underset{\sim}{X}}\{\bigcup_{s\in S(\alpha)} U(\underset{\sim}{y}_s) \cap D_i\} \le P_{\underset{\sim}{X}}\{\bigcup_{s\in S(\alpha)} U(\underset{\sim}{y}_s)\} = 0$$

from (4.1) and Theorem 4.2. Similarly, the third term vanishes, and hence

$$P_{\underset{\sim}{X}}\{\bigcup_{t\in T(\alpha)} U(\underset{\sim}{y}_t) \cap D_i\} = P_{\underset{\sim}{X}}\{\bigcup_{\ell\in L(\alpha)} U(\underset{\sim}{y}_\ell) \cap D_i\}.$$

Since, by hypothesis, $y_i = 0$ for all $\underset{\sim}{y} \in P_{\alpha 1}$, $U(\underset{\sim}{y})$ must be of the form

$$[y_1,1]\times\cdots\times[y_{i-1},1]\times[0,1]\times[y_{i+1},1]\times\cdots\times[y_n,1]$$

and so

$$P_{\underset{\sim}{X}}\{\bigcup_{\ell\in L(\alpha)} U(\underset{\sim}{y}_\ell)\cap D_i\}$$

$$= P\{\bigcup_{\ell\in L(\alpha)}\bigcap_{j\neq i}\{X_j \geq y_{\ell j}\}\cap\{X_i \geq \delta\}\}$$

$$= P\{\bigcup_{\ell\in L(\alpha)}\bigcap_{j\neq i}\{X_j \geq y_{\ell j}\}\}P\{X_i \geq \delta\} \text{ by independence.}$$

Thus,

$$(4.2) \qquad f_i(\alpha) = P\{\bigcup_{\ell\in L(\alpha)}\bigcap_{j\neq i}\{X_j \geq y_{\ell j}\}\}.$$

By a similar argument,

$$P_{\underset{\sim}{X}}\{\bigcup_{t\in T(\alpha)} U(\underset{\sim}{y}_t)\cap E_i\}$$

$$= P_{\underset{\sim}{X}}\{[\bigcup_{\ell\in L(\alpha)} U(\underset{\sim}{y}_\ell)\cup\bigcup_{s\in S(\alpha)} U(\underset{\sim}{y}_s)]\cap E_i\}$$

$$= P_{\underset{\sim}{X}}\{\bigcup_{\ell\in L(\alpha)} U(\underset{\sim}{y}_\ell)\cap E_i\}$$

$$= P_{\underset{\sim}{X}}\{\bigcup_{\ell\in L(\alpha)}\bigcap_{j\neq i}\{X_j \geq y_{\ell j}\}\cap\{X_i<\delta\}\}$$

$$= P_{\underset{\sim}{X}}\{\bigcup_{\ell\in L(\alpha)}\bigcap_{j\neq i}\{X_j \geq y_{\ell j}\}\}P\{X_i<\delta\}.$$

Thus,

$$(4.3) \qquad g_i(\alpha) = P\{\bigcup_{\ell\in L(\alpha)}\bigcap_{j\neq i}\{X_j \geq y_{\ell j}\}\}.$$

From (4.2) and (4.3), we see that $R_i(\alpha) = 0$ as claimed.

"<u>Only if</u>" Since, by hypothesis, $\nu\{U_\alpha\} > 0$ and since, by Lemma 3.1, $U_\alpha = \bigcup_{t\in T(\alpha)} U(\underset{\sim}{y}_t)$, we see that $\nu\{\bigcup_{t\in T(\alpha)} U(\underset{\sim}{y}_t)\} > 0$. Thus, by Lemma 4.1 (ii), there exists some $t' \in T(\alpha)$ such that $\nu\{U(\underset{\sim}{y}_{t'})\} > 0$. Write

$$P_{\alpha a} = \{\underset{\sim}{y}\in P_\alpha | \nu\{U(\underset{\sim}{y})\}>0\} = \{\underset{\sim}{y}_{t'}, t'\in T'(\alpha)\}$$

$$P_{\alpha b} = \{\underset{\sim}{y}\in P_\alpha | \nu\{U(\underset{\sim}{y})\}=0\} = \{\underset{\sim}{y}_w, w\in W(\alpha)\}, \text{ say.}$$

Then $P_{\alpha a}$ and $P_{\alpha b}$ form a partition of P_α. It is sufficient to show that if $R_i(\alpha) = 0$, then $y_i = 0$ for all $\underset{\sim}{y} \in P_{\alpha a}$.

Suppose that there exists a vector $\underset{\sim}{y} \in P_{\alpha a}$ such that $y_i \neq 0$. Since $\nu\{U(\underset{\sim}{y})\} > 0$ for all $\underset{\sim}{y} \in P_{\alpha a}$, it follows from Lemma 4.1(i) that $y_j \neq 1$ for all $j=1,2,\ldots,n$ and so $0 < y_i < 1$. Define the partition $P_{\alpha a} = P_{\alpha a1} \cup P_{\alpha a2} \cup P_{\alpha a3}$ where

$$P_{\alpha a1} = \{\underset{\sim}{y}\in P_\alpha | \nu\{U(\underset{\sim}{y})\}>0,\ y_i=0\} = \{\underset{\sim}{y}_\ell, \ell\in L(\alpha)\}$$

$$P_{\alpha a2} = \{\underset{\sim}{y}\in P_\alpha | \nu\{U(\underset{\sim}{y})\}>0,\ 0<y_i<\delta\} = \{\underset{\sim}{y}_s, s\in S(\alpha)\}$$

$$P_{\alpha a3} = \{\underset{\sim}{y}\in P_\alpha | \nu\{U(\underset{\sim}{y})\}>0,\ \delta\le y_i<1\} = \{\underset{\sim}{y}_m, m\in M(\alpha)\}, \text{ say.}$$

Then, clearly,

$$P\{\gamma(\underset{\sim}{X})\geq\alpha, X_i\geq\delta\} = P_{\underset{\sim}{X}}\{U_\alpha \cap D_i\}$$

$$= P_{\underset{\sim}{X}}[\{\bigcup_{\ell\in L(\alpha)} U(\underset{\sim}{y}_\ell) \cup \bigcup_{s\in S(\alpha)} U(\underset{\sim}{y}_s) \cup \bigcup_{m\in M(\alpha)} U(\underset{\sim}{y}_m)\} \cap D_i]$$

$$\geq P_{\underset{\sim}{X}}[\{\bigcup_{\ell\in L(\alpha)} U(\underset{\sim}{y}_\ell) \cup \bigcup_{s\in S(\alpha)} U(\underset{\sim}{y}_s)\} \cap D_i]$$

$$= P[\{\bigcup_{\ell\in L(\alpha)} \bigcap_{j=1}^{n} \{X_j\geq y_{\ell j}\} \cap \{X_i\geq\delta\}\} \cup \{\bigcup_{s\in S(\alpha)} \bigcap_{j=1}^{n} \{X_j\geq y_{sj}\} \cap \{X_i\geq\delta\}\}].$$

Since, by the definitions of $P_{\alpha a1}$ and $P_{\alpha a2}$, $\{X_i\geq y_{\ell i}\} \cap \{X_i\geq\delta\} =$ $\{X_i\geq y_{si}\} \cap \{X_i\geq\delta\} = \{X_i\geq\delta\}$, it follows from the independence of the X_i's that

$$P_{\underset{\sim}{X}}\{U_\alpha \cap D_i\} \geq P[\bigcup_{\ell\in L(\alpha)} \bigcap_{j\neq i} \{X_j\geq y_{\ell j}\} \cup \bigcup_{s\in S(\alpha)} \bigcap_{j\neq i} \{X_j\geq y_{sj}\}]P\{X_i\geq\delta\},$$

and hence

$$(4.4) \qquad f_i(\alpha) \geq P[\bigcup_{\ell\in L(\alpha)} \bigcap_{j\neq i} \{X_j\geq y_{\ell j}\} \cup \bigcup_{s\in S(\alpha)} \bigcap_{j\neq i} \{X_j\geq y_{sj}\}].$$

By a similar argument,

$$P\{\gamma(\underset{\sim}{X})\geq\alpha, X_i<\delta\} = P_{\underset{\sim}{X}}\{U_\alpha \cap E_i\}$$

$$= P_{\underset{\sim}{X}}[\{\bigcup_{\ell\in L(\alpha)} U(\underset{\sim}{y}_\ell) \cup \bigcup_{s\in S(\alpha)} U(\underset{\sim}{y}_s) \cup \bigcup_{m\in M(\alpha)} U(\underset{\sim}{y}_m)\} \cap E_i].$$

Recall that, by the definition of $P_{\alpha a3}$, $\delta \leq y_i$ for all $\underset{\sim}{y} \in P_{\alpha a3}$, so $\bigcup_{m\in M(\alpha)} U(\underset{\sim}{y}_m) \cap E_i = \emptyset$, and hence

$$(4.5) \qquad P_{\underset{\sim}{X}}\{U_\alpha \cap E_i\} = P_{\underset{\sim}{X}}\{\bigcup_{\ell\in L(\alpha)} \{U(\underset{\sim}{y}_\ell)\cap E_i\} \cup \bigcup_{s\in S(\alpha)} \{U(\underset{\sim}{y}_s)\cap E_i\}\}.$$

By the definitions of $P_{\alpha a2}$ and E_i, $U(\underset{\sim}{y}_s) \cap E_i$ is of the form $[y_{s1},1] \times \cdots \times [y_{s,i-1},1] \times [y_{si},\delta) \times [y_{s,i+1},1] \times \cdots \times [y_{sn},1]$ for all $s \in S(\alpha)$. Let

$$E_s = [y_{s1},1] \times \cdots \times [y_{s,i-1},1] \times [0,\delta) \times [y_{s,i+1},1] \times \cdots \times [y_{sn},1]$$

and

$$E_s' = [y_{s1},1] \times \cdots \times [y_{s,i-1},1] \times [0,y_{si}) \times [y_{s,i+1},1] \times \cdots \times [y_{sn},1]$$

for $s \in S(\alpha)$. Then $E_s = E_s' \cup [U(\underset{\sim}{y}_s) \cap E_i]$, and $\nu\{E_s'\} > 0$ and $\nu\{U(\underset{\sim}{y}_s) \cap E_i\} > 0$. It thus follows that

$$(4.6) \qquad P_{\underset{\sim}{X}}[\{\bigcup_{\ell \in L(\alpha)} U(\underset{\sim}{y}_\ell) \cap E_i\} \cup \bigcup_{s \in S(\alpha)} E_s]$$

$$= P_{\underset{\sim}{X}}[\{\bigcup_{\ell \in L(\alpha)} U(\underset{\sim}{y}_\ell) \cap E_i\} \cup \bigcup_{s \in S(\alpha)} E_s' \cup \{\bigcup_{s \in S(\alpha)} U(\underset{\sim}{y}_s) \cap E_i\}]$$

$$= P_{\underset{\sim}{X}}[\{\bigcup_{\ell \in L(\alpha)} U(\underset{\sim}{y}_\ell) \cap E_i\} \cup \{\bigcup_{s \in S(\alpha)} U(\underset{\sim}{y}_s) \cap E_i\}] + P_{\underset{\sim}{X}}\{\bigcup_{s \in S(\alpha)} E_s'\}$$

$$- P_{\underset{\sim}{X}}[(\{\bigcup_{\ell \in L(\alpha)} U(\underset{\sim}{y}_\ell) \cap E_i\} \cup \{\bigcup_{s \in S(\alpha)} U(\underset{\sim}{y}_s) \cap E_i\}) \cap \bigcup_{s \in S(\alpha)} E_s']$$

$$= P_{\underset{\sim}{X}}[\{\bigcup_{\ell \in L(\alpha)} U(\underset{\sim}{y}_\ell) \cap E_i\} \cup \{\bigcup_{s \in S(\alpha)} U(\underset{\sim}{y}_s) \cap E_i\}] + P_{\underset{\sim}{X}}\{\bigcup_{s \in S(\alpha)} E_s'\}$$

$$- P_{\underset{\sim}{X}}[\{\bigcup_{\ell \in L(\alpha)} U(\underset{\sim}{y}_\ell) \cap E_i\} \cap \bigcup_{s \in S(\alpha)} E_s'] \text{ since } U(\underset{\sim}{y}_s) \cap E_i \cap E_s' = \emptyset$$

for all $s \in S(\alpha)$.

<u>Claim</u>: $P_{\underset{\sim}{X}}\{\bigcup_{s \in S(\alpha)} E_s'\} - P_{\underset{\sim}{X}}[\{\bigcup_{\ell \in L(\alpha)} U(\underset{\sim}{y}_\ell) \cap E_i\} \cap \bigcup_{s \in S(\alpha)} E_s'] > 0.$

Proof of claim: We show, equivalently, that

$$P_{\underset{\sim}{X}}[\{\bigcup_{\ell\in L(\alpha)} U(\underset{\sim}{y}_\ell)\cap E_i\}^c \cap \{\bigcup_{s\in S(\alpha)} E_s'\}] > 0.$$

Since $\bigcup_{\ell\in L(\alpha)} U(\underset{\sim}{y}_\ell)\cap E_i \subset \bigcup_{\ell\in L(\alpha)} U(\underset{\sim}{y}_\ell) \subset U_\alpha$, it follows that

$\{\bigcup_{\ell\in L(\alpha)} U(\underset{\sim}{y}_\ell)\cap E_i\}^c \supset U_\alpha^c$, and hence

$$\nu\{\{\bigcup_{\ell\in L(\alpha)} U(\underset{\sim}{y}_\ell)\cap E_i\}^c \cap \bigcup_{s\in S(\alpha)} E_s'\} \geq \nu\{U_\alpha^c \cap \bigcup_{s\in S(\alpha)} E_s'\}$$

$$\geq \nu\{U_\alpha^c \cap E_s'\} \text{ for each } s\in S(\alpha).$$

Consider the vector $\underset{\sim}{y}' = (0_i, \underset{\sim}{y}_s) \in \Delta$ for $s \in S(\alpha)$; clearly $\underset{\sim}{y}' \in U_\alpha^c$. Since U_α is closed, by virtue of the continuity of γ, U_α^c is open, so there exists a vector $\underset{\sim}{z} \in U_\alpha^c$ such that $z_j > y_j'$ for $j\neq i$ and $0<z_i<y_{si}$. Define $E'' = [y_1', z_1]\times\cdots\times[y_n', z_n]$; clearly, $E'' \subset U_\alpha^c \cap E_s'$. Thus

$\nu\{U_\alpha^c \cap E_s'\} \geq \nu\{E''\} > 0$, i.e. $\nu\{\{\bigcup_{\ell\in L(\alpha)} U(\underset{\sim}{y}_\ell)\cap E_i\}^c \cap \bigcup_{s\in S(\alpha)} E_s'\} > 0$, so that,

by Theorem 4.2, $P_{\underset{\sim}{X}}[\{\bigcup_{\ell\in L(\alpha)} U(\underset{\sim}{y}_\ell)\cap E_i\}^c \cap \bigcup_{s\in S(\alpha)} E_s'] > 0$.

This completes the proof of the claim. □

It now follows from (4.6) that

$$P_{\underset{\sim}{X}}[\{\bigcup_{\ell\in L(\alpha)} U(\underset{\sim}{y}_\ell)\cap E_i\} \cup \bigcup_{s\in S(\alpha)} E_s]$$

$$> P_{\underset{\sim}{X}}[\{\bigcup_{\ell\in L(\alpha)} U(\underset{\sim}{y}_\ell)\cap E_i\} \cup \{\bigcup_{s\in S(\alpha)} U(\underset{\sim}{y}_\ell)\cap E_i\}].$$

Observe that

$$P_{\underset{\sim}{X}}[\{\bigcup_{\ell\in L(\alpha)} U(\underset{\sim}{y}_\ell)\cap E_i\} \cup \bigcup_{s\in S(\alpha)} E_s]$$

$$= P[\{\bigcup_{\ell\in L(\alpha)} \bigcap_{j\neq i}\{X_j\geq y_{\ell j}\} \cap \{X_i<\delta\}\} \cup \{\bigcup_{s\in S(\alpha)} \bigcap_{j\neq i}\{X_j\geq y_{sj}\} \cap \{X_i<\delta\}\}]$$

$$= P[\bigcup_{\ell\in L(\alpha)} \bigcap_{j\neq i}\{X_j\geq y_{\ell j}\} \cup \bigcup_{s\in S(\alpha)} \bigcap_{j\neq i}\{X_j\geq y_{sj}\}]P\{X_i<\delta\}$$

by independence, and so from (4.5)

$$(4.7) \qquad g_i(\alpha) < P[\bigcup_{\ell\in L(\alpha)} \bigcap_{j\neq i} \{X_j\geq y_{\ell j}\} \cup \bigcup_{s\in S(\alpha)} \bigcap_{j\neq i}\{X_j\geq y_{sj}\}].$$

Thus, by (4.4) and (4.7), $R_i(\alpha) > 0$, thereby contradicting the assumption that $R_i(\alpha) = 0$.

This completes the proof. $\square$

<u>Corollary 4.5</u>

Let γ be a continuous CSF such that $\nu\{U_\alpha\} > 0$ for all $\alpha \in (0,1)$ and suppose that $X_1,\ldots,X_n$ are independent, absolutely continuous random variables. Then $R_i(\alpha) > 0$ for $\alpha \in (0,1)$ if and only if $y_i \neq 0$ for some $\underset{\sim}{y} \in P_\alpha$ for which $\nu\{U(\underset{\sim}{y})\} > 0$.

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SECURITY CLASSIFICATION OF THIS PAGE

REPORT DOCUMENTATION PAGE

1a. REPORT SECURITY CLASSIFICATION UNCLASSIFIED	1b. RESTRICTIVE MARKINGS
2a. SECURITY CLASSIFICATION AUTHORITY	3. DISTRIBUTION/AVAILABILITY OF REPORT Approved for public release; distribution unlimited.
2b. DECLASSIFICATION/DOWNGRADING SCHEDULE	
4. PERFORMING ORGANIZATION REPORT NUMBER(S)	5. MONITORING ORGANIZATION REPORT NUMBER(S) **AFOSR-TR- 86-0035**
6a. NAME OF PERFORMING ORGANIZATION STATE UNIVERSITY OF NEW YORK AT STONY BROOK 6b. OFFICE SYMBOL (If applicable)	7a. NAME OF MONITORING ORGANIZATION Air Force Office of Scientific Research
6c. ADDRESS (City, State and ZIP Code) DEPT. OF APPLIED MATHEMATICS & STATISTICS STATE UNIVERSITY OF NEW YORK AT STONY BROOK, STONY BROOK, N.Y. 11794	7b. ADDRESS (City, State and ZIP Code) Directorate of Mathematical & Information Sciences, Bolling AFB DC 20332-6448
8a. NAME OF FUNDING/SPONSORING ORGANIZATION AFOSR 8b. OFFICE SYMBOL (If applicable) NM	9. PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER AFOSR 84-0243
8c. ADDRESS (City, State and ZIP Code) Bolling AFB DC 20332-6448	10. SOURCE OF FUNDING NOS. PROGRAM ELEMENT NO. 61102F; PROJECT NO. 2304; TASK NO. A5; WORK UNIT NO.

11. TITLE (Include Security Classification)
Reliability Importance for Continuum Structure Functions

12. PERSONAL AUTHOR(S)
Chul Kim and Laurence A. Baxter

13a. TYPE OF REPORT	13b. TIME COVERED	14. DATE OF REPORT (Yr., Mo., Day)	15. PAGE COUNT
Technical	FROM ______ TO ______	1985	24

16. SUPPLEMENTARY NOTATION

17. COSATI CODES			18. SUBJECT TERMS (Continue on reverse if necessary and identify by block number)
FIELD	GROUP	SUB. GR.	

19. ABSTRACT (Continue on reverse if necessary and identify by block number)

A continuum structure function is a nondecreasing mapping from the unit hypercube to the unit interval. A definition of the reliability importance, $R_i(\alpha)$ say, of component i at level α $(0<\alpha\leq 1)$ is proposed. Some properties of this function are deduced, in particular conditions under which $\lim_{\alpha\to 0} R_i(\alpha) = \lim_{\alpha\to 1} R_i(\alpha) = 0$ and conditions under which $R_i(\alpha)$ is positive $(0<\alpha<1)$.

20. DISTRIBUTION/AVAILABILITY OF ABSTRACT UNCLASSIFIED/UNLIMITED ☒ SAME AS RPT. ☐ DTIC USERS ☐	21. ABSTRACT SECURITY CLASSIFICATION UNCLASSIFIED
22a. NAME OF RESPONSIBLE INDIVIDUAL Major B. W. Woodruff	22b. TELEPHONE NUMBER (Include Area Code) (202) 767-5027 22c. OFFICE SYMBOL NM

DD FORM 1473, 83 APR EDITION OF 1 JAN 73 IS OBSOLETE. UNCLASSIFIED

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